Bitcoin realized price for various probability measures on UTXO set

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2022 May

Bitcoin realized price for various probability measures (p.m.) on UTXO set U is considered. Realized cap (price) is a fundamental characteristic and it was introduced in the article "Introducing Realized Capitalization" by CoinMetrics in December 2018. According to the original definition, and based on our notations in the article "A note on one Bitcoin statistical project", realized price is a mathematical expectation of a tx output's created price (a price level when an output was created in the chain) for a non-uniform p.m. with a weight (mass) $\mathbf{v}$:

$$r_{\mathcal{P}_{\mathbf{v}}} = \left(\sum_{u \in U} \mathbf{v}(u) \right)^{-1} \sum_{u \in U} p(\mathbf{h}(u)) \mathbf{v}(u),$$

where $\mathbf{v}(u)$ is a value of the output $u \in U$ in Satoshi, $p(\mathbf{h}(u))$ is a price level at the point $\mathbf{h}(u)$ when the output was created.

A uniform p.m. is a widely used measure in probabilty theory. Realized price for a uniform p.m. is:

$$r_{\mathcal{P}} = |U|^{-1} \sum_{u \in U} p(\mathbf{h}(u)),$$

for a non-uniform p.m. with a weight $\mathfrak{h}\mathbf{v}$ (this product of two terms is applied for the "coin days destroyed" characteristic):

$$r_{\mathcal{P}_{\mathfrak{h}\mathbf{v}}} = \left(\sum_{u \in U} \mathfrak{h}(u) \mathbf{v}(u) \right)^{-1} \sum_{u \in U} p(\mathbf{h}(u)) \mathfrak{h}(u) \mathbf{v}(u),$$

where $\mathfrak{h}(u) = h - \mathbf{h}(u)$ is a lifespan, and h is a current point. Similarly, we can build $r_{\mathcal{P}_{\mathfrak{h}}}$ for a p.m. $\mathcal{P}_{\mathfrak{h}}$.

More specifically, "long-term" and "short-term" variants of p.m. are defined. Suppose $\Delta \in \mathbb{N} \cup \{0\}$ is a threshold parameter, and $\beta = \beta(u) = [\mathfrak{h}(u) < \Delta] \in \{0, 1\}$ is an indicator function. For the "short-term" variant we have that if the lifespan $\mathfrak{h}(u)$ of the output $u \in U$ is equal or more than Δ, then a probability of such output is equal to zero. In experiments the threshold is set to 210000 blocks (about four years). In this case realized price for a probability measure with a weight $\beta\mathbf{f}$ is:

$$r_{\mathcal{P}_{\beta\mathbf{f}}} = \left(\sum_{u \in U} \beta\mathbf{f}(u) \right)^{-1} \sum_{u \in U} p(\mathbf{h}(u)) \beta\mathbf{f}(u),$$

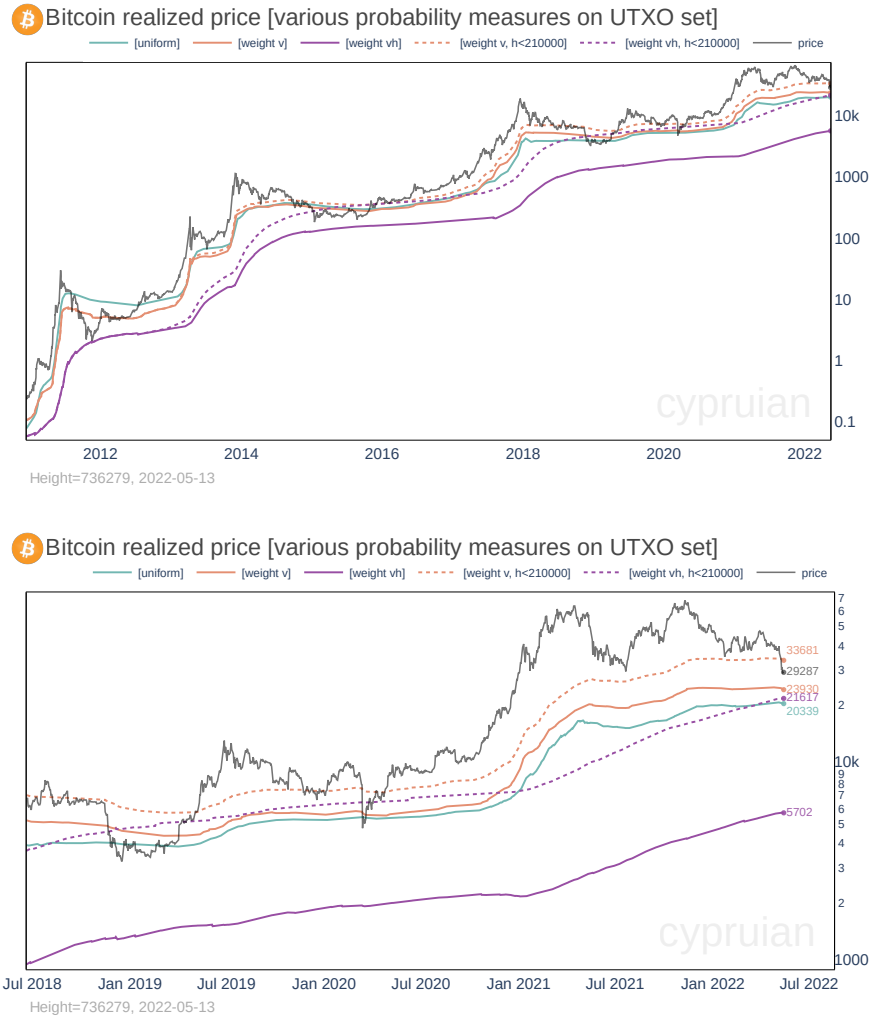

Figure 1: Bitcoin realized price for various p.m.

where $\beta\mathbf{f}$ is some weight, and e.g., $f \in \{\mathbf{v}, \mathbf{v}\mathfrak{h}, \mathfrak{h}\}$. If $1-\beta$ is used instead of β with the function $\mathbf{f}$, it leads to the "long-term" variant.

Bitcoin realized price r for various probability measures is presented in the figure 1.